NAG C Library Function Document

nag_zgebrd (f08ksc)

1 Purpose

nag_zgebrd (f08ksc) reduces a complex m by n matrix to bidiagonal form.

2 Specification

```
void nag_zgebrd (Nag_OrderType order, Integer m, Integer n, Complex a[],
      Integer pda, double d[], double e[], Complex tauq[], Complex taup[],
      NagError *fail)
```

3 Description

nag_zgebrd (f08ksc) reduces a complex m by n matrix A to real bidiagonal form B by a unitary transformation: $A = QBP^H$, where Q and P^H are unitary matrices of order m and n respectively.

If $m \geq n$, the reduction is given by:

$$A = Q\begin{pmatrix} B_1 \\ 0 \end{pmatrix}P^H = Q_1B_1P^H,$$

where B_1 is a real n by n upper bidiagonal matrix and Q_1 consists of the first n columns of Q.

If $m < n$, the reduction is given by

$$A = Q(\,B_1 \quad 0\,)P^H = QB_1P_1^H,$$

where B_1 is a real m by m lower bidiagonal matrix and P_1^H consists of the first m rows of P^H.

The unitary matrices Q and P are not formed explicitly but are represented as products of elementary reflectors (see the f08 Chapter Introduction for details). Functions are provided to work with Q and P in this representation (see Section 8).

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

1: **order** – Nag_OrderType *Input*

On entry: the **order** parameter specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = **Nag_RowMajor**. See Section 2.2.1.4 of the Essential Introduction for a more detailed explanation of the use of this parameter.

Constraint: **order** = **Nag_RowMajor** or **Nag_ColMajor**.

2: **m** – Integer *Input*

On entry: m, the number of rows of the matrix A.

Constraint: **m** ≥ 0.

3: **n** – Integer *Input*

On entry: n, the number of columns of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

4: $\mathbf{a}[dim]$ – Complex *Input/Output*

Note: the dimension, dim, of the array **a** must be at least $\max(1, \mathbf{pda} \times \mathbf{n})$ when **order** = **Nag_ColMajor** and at least $\max(1, \mathbf{pda} \times \mathbf{m})$ when **order** = **Nag_RowMajor**.

If **order** = **Nag_ColMajor**, the (i, j)th element of the matrix A is stored in $\mathbf{a}[(j-1) \times \mathbf{pda} + i - 1]$ and if **order** = **Nag_RowMajor**, the (i, j)th element of the matrix A is stored in $\mathbf{a}[(i-1) \times \mathbf{pda} + j - 1]$.

On entry: the m by n matrix A.

On exit: if $m \geq n$, the diagonal and first super-diagonal are overwritten by the upper bidiagonal matrix B, elements below the diagonal are overwritten by details of the unitary matrix Q and elements above the first super-diagonal are overwritten by details of the unitary matrix P.

If $m < n$, the diagonal and first sub-diagonal are overwritten by the lower bidiagonal matrix B, elements below the first sub-diagonal are overwritten by details of the unitary matrix Q and elements above the diagonal are overwritten by details of the unitary matrix P.

5: **pda** – Integer *Input*

On entry: the stride separating matrix row or column elements (depending on the value of **order**) in the array **a**.

Constraints:

if **order** = **Nag_ColMajor**, $\mathbf{pda} \geq \max(1, \mathbf{m})$;
if **order** = **Nag_RowMajor**, $\mathbf{pda} \geq \max(1, \mathbf{n})$.

6: $\mathbf{d}[dim]$ – double *Output*

Note: the dimension, dim, of the array **d** must be at least $\max(1, \min(\mathbf{m}, \mathbf{n}))$.

On exit: the diagonal elements of the bidiagonal matrix B.

7: $\mathbf{e}[dim]$ – double *Output*

Note: the dimension, dim, of the array **e** must be at least $\max(1, \min(\mathbf{m}, \mathbf{n}) - 1)$.

On exit: the off-diagonal elements of the bidiagonal matrix B.

8: $\mathbf{tauq}[dim]$ – Complex *Output*

Note: the dimension, dim, of the array **tauq** must be at least $\max(1, \min(\mathbf{m}, \mathbf{n}))$.

On exit: further details of the unitary matrix Q.

9: $\mathbf{taup}[dim]$ – Complex *Output*

Note: the dimension, dim, of the array **taup** must be at least $\max(1, \min(\mathbf{m}, \mathbf{n}))$.

On exit: further details of the unitary matrix P.

10: **fail** – NagError * *Output*

The NAG error parameter (see the Essential Introduction).

6 Error Indicators and Warnings

NE_INT

On entry, **m** = $\langle value \rangle$.
Constraint: $\mathbf{m} \geq 0$.

On entry, **n** = $\langle$*value*$\rangle$.
Constraint: **n** ≥ 0.

On entry, **pda** = $\langle$*value*$\rangle$.
Constraint: **pda** > 0.

NE_INT_2

On entry, **pda** = $\langle$*value*$\rangle$, **m** = $\langle$*value*$\rangle$.
Constraint: **pda** $\geq \max(1, \mathbf{m})$.

On entry, **pda** = $\langle$*value*$\rangle$, **n** = $\langle$*value*$\rangle$.
Constraint: **pda** $\geq \max(1, \mathbf{n})$.

NE_ALLOC_FAIL

Memory allocation failed.

NE_BAD_PARAM

On entry, parameter $\langle$*value*$\rangle$ had an illegal value.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

7 Accuracy

The computed bidiagonal form B satisfies $QBP^H = A + E$, where

$$\|E\|_2 \leq c(n)\epsilon\|A\|_2,$$

$c(n)$ is a modestly increasing function of n, and ϵ is the ***machine precision***.

The elements of B themselves may be sensitive to small perturbations in A or to rounding errors in the computation, but this does not affect the stability of the singular values and vectors.

8 Further Comments

The total number of real floating-point operations is approximately $16n^2(3m - n)/3$ if $m \geq n$ or $16m^2(3n - m)/3$ if $m < n$.

If $m \gg n$, it can be more efficient to first call nag_zgeqrf (f08asc) to perform a QR factorization of A, and then to call nag_zgebrd (f08ksc) to reduce the factor R to bidiagonal form. This requires approximately $8n^2(m + n)$ floating-point operations.

If $m \ll n$, it can be more efficient to first call nag_zgelqf (f08avc) to perform an LQ factorization of A, and then to call nag_zgebrd (f08ksc) to reduce the factor L to bidiagonal form. This requires approximately $8m^2(m + n)$ operations.

To form the unitary matrices P^H and/or Q, this function may be followed by calls to nag_zungbr (f08ktc):

to form the m by m unitary matrix Q

```
nag_zungbr (order,Nag_FormQ,m,m,n,&a,pda,tauq,&fail)
```

but note that the second dimension of the array **a** must be at least **m**, which may be larger than was required by nag_zgebrd (f08ksc);

to form the n by n unitary matrix P^H

```
nag_zungbr (order,Nag_FormP,n,n,m,&a,pda,taup,&fail)
```

but note that the first dimension of the array **a**, specified by the parameter **pda**, must be at least **n**, which may be larger than was required by nag_zgebrd (f08ksc).

To apply Q or P to a complex rectangular matrix C, this function may be followed by a call to nag_zunmbr (f08kuc).

The real analogue of this function is nag_zgebrd (f08ksc).

9 Example

To reduce the matrix A to bidiagonal form, where

$$A = \begin{pmatrix} 0.96 - 0.81i & -0.03 + 0.96i & -0.91 + 2.06i & -0.05 + 0.41i \\ -0.98 + 1.98i & -1.20 + 0.19i & -0.66 + 0.42i & -0.81 + 0.56i \\ 0.62 - 0.46i & 1.01 + 0.02i & 0.63 - 0.17i & -1.11 + 0.60i \\ -0.37 + 0.38i & 0.19 - 0.54i & -0.98 - 0.36i & 0.22 - 0.20i \\ 0.83 + 0.51i & 0.20 + 0.01i & -0.17 - 0.46i & 1.47 + 1.59i \\ 1.08 - 0.28i & 0.20 - 0.12i & -0.07 + 1.23i & 0.26 + 0.26i \end{pmatrix}.$$

9.1 Program Text

```
/* nag_zgebrd (f08ksc) Example Program.
 *
 * Copyright 2001 Numerical Algorithms Group.
 *
 * Mark 7, 2001.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf08.h>

int main(void)
{
  /* Scalars */
  Integer  i, j, m, n, pda, d_len, e_len, tauq_len, taup_len;
  Integer  exit_status=0;
  NagError fail;
  Nag_OrderType order;
  /* Arrays */
  Complex *a=0, *taup=0, *tauq=0;
  double  *d=0, *e=0;

#ifdef NAG_COLUMN_MAJOR
#define A(I,J) a[(J-1)*pda + I - 1]
  order = Nag_ColMajor;
#else
#define A(I,J) a[(I-1)*pda + J - 1]
  order = Nag_RowMajor;
#endif

  INIT_FAIL(fail);
  Vprintf("f08ksc Example Program Results\n");

  /* Skip heading in data file */
  Vscanf("%*[^\n] ");
  Vscanf("%ld%ld%*[^\n] ", &m, &n);
#ifdef NAG_COLUMN_MAJOR
  pda = m;
#else
  pda = n;
#endif
  d_len = MIN(m,n);
  e_len = MIN(m,n)-1;
  tauq_len = MIN(m,n);
  taup_len = MIN(m,n);

  /* Allocate memory */
  if ( !(a = NAG_ALLOC(m * n, Complex)) ||
       !(d = NAG_ALLOC(d_len, double)) ||
```

```
        !(e = NAG_ALLOC(e_len, double)) ||
        !(taup = NAG_ALLOC(taup_len, Complex)) ||
        !(tauq = NAG_ALLOC(tauq_len, Complex)) )
    {
      Vprintf("Allocation failure\n");
      exit_status = -1;
      goto END;
    }

  /* Read A from data file */
  for (i = 1; i <= m; ++i)
    {
      for (j = 1; j <= n; ++j)
        Vscanf(" ( %lf , %lf )", &A(i,j).re, &A(i,j).im);
    }
  Vscanf("%*[^\n] ");

  /* Reduce A to bidiagonal form */
  f08ksc(order, m, n, a, pda, d, e, tauq, taup, &fail);
  if (fail.code != NE_NOERROR)
    {
      Vprintf("Error from f08ksc.\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }

  /* Print bidiagonal form */
  Vprintf("\nDiagonal\n");
  for (i = 1; i <= MIN(m,n); ++i)
    Vprintf("%9.4f%s", d[i-1], i%8==0 ?"\n":" ");
  if (m >= n)
    Vprintf("\nSuper-diagonal\n");
  else
    Vprintf("\nSub-diagonal\n");
  for (i = 1; i <= MIN(m,n) - 1; ++i)
    Vprintf("%9.4f%s", e[i-1], i%8==0 ?"\n":" ");
  Vprintf("\n");

 END:
  if (a) NAG_FREE(a);
  if (d) NAG_FREE(d);
  if (e) NAG_FREE(e);
  if (taup) NAG_FREE(taup);
  if (tauq) NAG_FREE(tauq);

  return exit_status;
}
```

9.2 Program Data

```
f08ksc Example Program Data
  6  4                                                      :Values of M and N
 ( 0.96,-0.81) (-0.03, 0.96) (-0.91, 2.06) (-0.05, 0.41)
 (-0.98, 1.98) (-1.20, 0.19) (-0.66, 0.42) (-0.81, 0.56)
 ( 0.62,-0.46) ( 1.01, 0.02) ( 0.63,-0.17) (-1.11, 0.60)
 (-0.37, 0.38) ( 0.19,-0.54) (-0.98,-0.36) ( 0.22,-0.20)
 ( 0.83, 0.51) ( 0.20, 0.01) (-0.17,-0.46) ( 1.47, 1.59)
 ( 1.08,-0.28) ( 0.20,-0.12) (-0.07, 1.23) ( 0.26, 0.26)   :End of matrix A
```

9.3 Program Results

```
f08ksc Example Program Results

Diagonal
  -3.0870    2.0660    1.8731    2.0022
Super-diagonal
   2.1126    1.2628   -1.6126
```